Section I:

1. a) Derive the Maxwell's equations

$$\nabla \times \vec{E} = -j\omega\vec{B}$$
$$\nabla \cdot \vec{D} = \rho$$
$$\nabla \times \vec{H} = \vec{J} + j\omega\vec{D}$$
$$\nabla \cdot \vec{B} = 0$$

from Coulomb's law, Ampere's law, Faraday's law, and the principle of conservation of electric charge. (10 points)

b) Derive the wave equations of $\vec{E}$ and $\vec{H}$. (5 points)

c) Solve these two wave equations to obtain time-harmonic plane waves in a lossless medium. Assume that the waves propagate in the *z*-direction. (5 points)

2. The effective complex dielectric constants of walls in buildings are investigated for wireless communication applications. The relative dielectric constant of the reinforced concrete wall of a building is found to be $\varepsilon_r = 6.7 - j1.2$ at 900 MHz and $\varepsilon_r = 6.2 - j0.69$ at 1.8 GHz, respectively. (a) Find the appropriate thickness of the concrete wall to cause a 10 dB attenuation in the filed strength of the 900 MHz signal traveling over its thickness. Neglect the reflections from the surfaces of the wall. (10%) (b) Repeat the same calculations at 1.8 GHz. (5 points)

3. The sum of the electric fields of two time-harmonic (sinusoidal) electromagnetic waves propagating in opposite directions in air is given as

$$\vec{E}(z,t) = \hat{x}95\sin(\beta z)\sin(21\times 10^9 \pi t) \quad \text{mV/m}$$

a) Find the constant β. (5 points)

b) Find the corresponding $\vec{H}$. (5 points)

c) Assuming that this wave may be regarded as a sum of two uniform plane waves, determine the direction of propagation of the two component waves. (5 points)

Section II:

4. (a) What is an antenna? What does it possess basic functions? (10 points)

(b) What is the Hertzian dipole antenna? Give your illustrations and derive what are electric and magnetic fields? (20 points)

5. Suppose that a plane electromagnetic wave of frequency $\omega/2\pi$ and amplitude E_0 is normally incident on the flat surface of a semi-infinite metal of conductivity σ. Assume the frequency is low so that the displacement current inside the metal can be neglected. The magnetic permeability of the metal μ=1.

(a) Using Maxwell's equations, derive expressions for the components of the electric and magnetic fields inside the conductor which are parallel to the surface. What is the characteristic penetration depth of the field? (10 points)

(b) What is the ratio of the magnetic field amplitude to the electric field amplitude inside the metal?

(5 points)

(c) What is the power per unit area transmitted into the metal? (5 points)

Section III

6. For 2D parallel plate metal waveguide with thickness *a* filled with ε_d, μ_d, consider the TE waveguide mode propagating in the z direction with field components H_x, H_z, E_y (10 points)

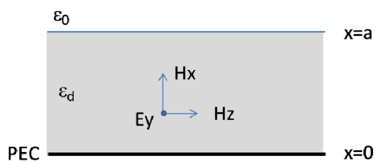
(a) Using the wave equation and the boundary conditions for Ey (PEC plate at x=0 and x=a), derive the field component Ey and then using Maxwell curl equation to obtain H_x, H_z of a TE parallel plate waveguide

(b) Find the cut-off frequency ω_{cm} (m is the waveguide mode), the propagation constant β_z, and its impedance η?

7. Following problem 1, now remove both the upper and bottom metal plate, it becomes a dielectric slab waveguide with ε_d, μ_d. (20 points)

(a) Find the dispersion relation of the TE even and odd mode.

(b) If only the upper metal plate is removed, what are the allowed TE modes and the cutoff frequency?



8. Rectangular Waveguide TE_{10} mode (20 points)

For a rectangular perfect conducting waveguide with size a and b (a>b) as shown in the figure below, assume the wave is guiding in the z-direction with z dependence as $e^{-\gamma z}$. For the Transverse electric(TE) wave, E_z=0,

(a) Solve for H_z and write down other components E_x, E_y, H_x, H_y

(b) what are the *cutoff frequency* ω_c, *phase velocity* v_p and *impedance* η for the TE_{10} mode

(c) By closing both ends of the rectangular waveguide with length d=a, the rectangular waveguide become a resonator. Find the lowest order resonant frequency and their corresponding field components E_x, E_y, H_x, H_y, H_z.

(d) If the cavity resonator with equal size a in all dimension (a=b=d) is cut by half as a right-angle triangular cavity resonator, surface x=y is also covered with PEC. What is the resonate frequency of the first TE cavity mode? What are the E_x E_y field components? (Hint: this can be considered as a superposition of two rectangular cavity modes, such that all the parallel E fields at the edges of the triangle vanish.)