## Part I (50 points)

The permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12} F/m$

The permeability of free space $\mu_0 = 4\pi \times 10^{-17} H/m$

The velocity of electromagnetic wave in free space $c = 3 \times 10^8 m/s$

1. The electric field in a high power laser is described by
   $E = (0.700 \times 10^6 V/m)\sin(5.93 \times 10^6 x - \omega t)$
   Find (a) the amplitude of the magnetic field produced, (2%) (b) the wavelength of the laser,(2%) (c) the intensity of the laser,(4%) and (d) the power delivered by the laser.(4%)
2. Assuming that permittivity ε is constant, derive wave equation in the form of the electric field from Maxwell's equation. (8%)
3. Show that the tangential component of the electric field is continuous across any interface from Maxwell's equation. Based on the abovementioned result, illustrate Snell's law. (10%)
4. An elliptically polarized beam propagating in the z direction is given by
   $\mathbf{E}(z,t) = \hat{\mathbf{x}}E_x + \hat{y}E_y = \mathrm{Re}[\hat{\mathbf{x}}C_x e^{i(\omega t - kz)} + \hat{y}C_y e^{i(\omega t - kz + \zeta)}]$.
   **(a)** Show that an elliptically polarized light will revolve in a clockwise direction if sinζ<0. This polarization is also called as the left-hand elliptical polarization. (10%)
   **(b)** Decompose elliptically polarized light in to a linear superposition of right-hand and left-hand circularly polarized states $\mathbf{R}$ and $\mathbf{L}$. i.e., find r and l such that $\mathbf{E} = r\mathbf{R} + l\mathbf{L}$. (10%)

## Part II (50 points)

1. A uniform plane wave in air with the following phase expression for electric intensity

$$\vec{E}_i(y,z) = 5\left(\hat{y} + \hat{z}\sqrt{3}\right)e^{j6\left(\sqrt{3}y - z\right)}$$

is incident on a perfectly conducting plane at $z = 0$. (25%)

(a) Find the frequency and wavelength of the wave.

(b) Write the instantaneous expression for the incident fields, $\boldsymbol{E_i}(y,z;t)$ and $\boldsymbol{H_i}(y,z;t)$, using a cosine reference.

(c) Determine the angle of incidence.

(d) Find $E_r(y,z)$ and $H_r(y,z)$ of the reflected wave.

(e) Find the time-averaged Poynting vector in air.

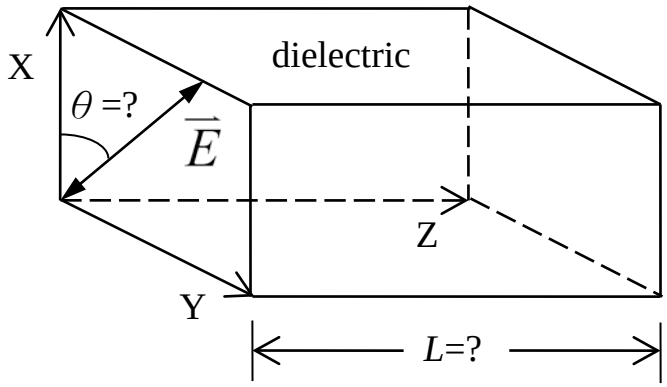
(f) Calculate the induced current on the conducting plane.

2. Regarding the polarization of a plane wave: (25%)

(a) Prove that a linearly polarized plane wave can be resolved into a right-hand circularly polarized wave and a left-circularly polarized wave of equal amplitude.

(b) Prove that an elliptically polarized plane wave can be resolved into right-hand circularly polarized wave and a left-circularly polarized wave.

(c) A plane wave at a wavelength of 1000 nm is linearly polarized at an angle $\theta$ with respect to the *x*-axis in air and propagates along the *z*-direction, as shown below. A dielectric with thickness *L* is placed in the path of wave propagation, which is characterized with a refractive index of 1.55 along the *x*-axis and a refractive index of 1.54 along the *y*-axis. If the plane wave becomes circularly polarized after passing through the dielectric, determine the angle $\theta$ and the thickness *L*. Note that the propagation constant $k = wn/c$ in a lossless medium, where *n* is the refractive index and *c* is the speed of light.



## Part III (50 points)

1. (a) Design an air-filled C-band (4—8 GHz) rectangular waveguide such that the center frequency of this band ($f$ = 6 GHz) is at least 25% higher than the cutoff frequency of the $TE_{10}$ mode and at least 25% lower than the cutoff frequency of the next higher mode, so that the dominate mode of propagation is $TE_{10}$. (5%) (b) Find the total time-average power for the $TE_{10}$ mode. (5%)
2. Find the filed solutions of TE and TM modes for a rectangular waveguide. (10%)
3. A static magnetic field can be used to produce electron beam deflection in a cathode-ray tube. As shown in following Figure, the magnetic field $\vec{B} = \hat{y}B_0$ is approximately uniform over the length of the deflection coil and is directed normal the paper. Electrons carrying charge $q_e$ each emerge from the cathode with an initial velocity $\vec{v} = \hat{z}v_0$. Determine the deflection d on the screen in terms of the given parameters. (10%)
4. The length of a Hertzian dipole, whose current *I* may be considered to be uniform at all points over its length, is denoted to be *dl*. Note that we have chosen to locate the Hertzian dipole at the origin.

   (a) Determine the phasor retarded vector potential $\vec{A}(\vec{r})$ in the spherical coordinate. (5%)

   (b) Find the instantaneous expression for the magnetic field $\vec{H}(\vec{r},t)$ of the electromagnetic radiation from this time-harmonic current. (5%)

   (c) Find the instantaneous expression for the electric field $\vec{E}(\vec{r},t)$ of the electromagnetic radiation from this time-harmonic current. (5%)

   (d) Discuss the electromagnetic fields at near-zone field, defined as $\beta r << 1$, and far-zone field, defined as $\beta r >> 1$, where $\beta$ is the wave number of the electromagnetic radiation. (5%)