Part I: 50 points, please describe your answers as complete as possible

1. Please derives (a) (10 pts) the equations of the dispersion relation $\omega = \omega(\mathbf{k}) = \omega(k_1, k_2, k_3)$ for a uniaxial crystal ($n_1 = n_2 = n_o$ and $n_3 = n_e$) and (b) (5 pts), according to the obtained answer in (a), plots the corresponding double-sheeted **k**-surface in the **k**-space with three normalized coordinate axes of k_1/k_0, k_2/k_0, and k_3/k_0, where $k_0 = \omega/c_0$, and indicates the intercepts at these axes with principal refractive indices $n_1 = (\varepsilon_1/\varepsilon_0)^{1/2}$, $n_2 = (\varepsilon_2/\varepsilon_0)^{1/2}$, and $n_3 = (\varepsilon_3/\varepsilon_0)^{1/2}$.

2. Determine the direction of propagation in quartz ($n_e = 1.553$ and $n_o = 1.544$) at which the angle between the wavevector **k** and the Poynting vector **S** (which is also the direction of ray propagation) is maximum. (8 pts)

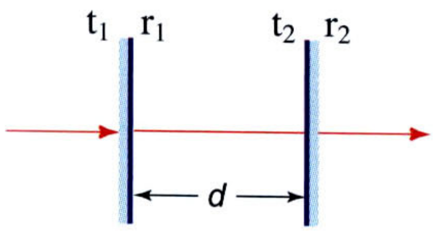
3. Consider a mirror Fabry-Perot etalon (as shown below) with two lossless partially reflective mirrors of amplitude transmittances t_1 and t_2, and amplitude reflectances r_1 and r_2, separated by a distance d filled with a medium of refractive index n. Prove the intensity transmittance T of the etalon as the following form (10 pts)

$$T(\nu) = \frac{T_{\max}}{1 + (2F/\pi)^2 \sin^2(\pi\nu/\nu_F)},$$

where,

$$T_{\max} = \frac{|t_1 t_2|^2}{(1-|r_1 r_2|)^2} = \frac{(1-|r_1|^2)(1-|r_2|^2)}{(1-|r_1 r_2|)^2},$$

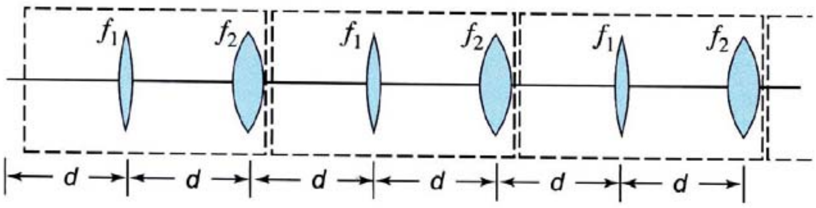
$$F = \frac{\pi\sqrt{|r_1 r_2|}}{1-|r_1 r_2|}, \text{ and } \quad \nu_F = \frac{c}{2d}.$$



4. Transmission of a Gaussian Beam Through a Graded-Index Slab. The ABCD matrix of a SELFOC graded-index slab with quadratic refractive index $n(y) \approx n_0(1-\frac{1}{2}\alpha^2 y^2)$ and length d is A = $\cos\alpha d$, B = $(1/\alpha)\sin\alpha d$, C = $-\alpha\sin\alpha d$, D = $\cos\alpha d$ for paraxial rays along the z direction. A Gaussian beam of wavelength λ_0, waist radius W_0 in free space, and axis in the z direction enters the slab at its waist. Use the ABCD law to determine an expression for the beam width in the y direction as a function of d. (10 pts)

5. Examine the trajectories of paraxial rays through a periodic system comprising a sequence of lens pairs with alternating focal lengths, f_1 and f_2, as shown below. Show that the ray trajectory is bounded (stable) if (7 pts)

$$0 \le \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right) \le 1$$



Part II: 50 points, please describe your answers as complete as possible

1. Explain why the linear term of the refractive index $n(E)$ of an electro-optic medium vanishes for an centrosymmetric material. (10 pts)

2. Linear absorption in a detector happens when one photon creates one electron-hole pair in the detector. In this case the resulting photocurrent is proportional to the incident optical power. But under certain conditions two photons may be absorbed in a detector generating one electron-hole pair and in this case the photocurrent is proportional to the square of the optical power. This phenomenon is called two-photon absorption (TPA). What is the order of the nonlinear optical effect responsible for TPA? (5 pts) Explain your answer. (10 pts)

3. Consider a laser amplifier with power gain coefficient $2\alpha_m(z)$ varying along the amplifier's length, and with a constant unsaturated background loss $2\alpha_0$. The gain is homogeneously broadened with saturation intensity $I_{sat} = 2\text{kW/cm}^2$. The amplifier length is $L = 100$ cm.

a. Write down the differential equation for the laser power inside the amplifier as a function of z. Neglect ASE in this part. (5 pts)

b. Considering the case of uniform gain $2\alpha_{m0}(z) = 0.1\ \text{cm}^{-1}$ and $\alpha_0 = 0$.
Find the small signal gain, by considering sufficiently small input. (5 pts)

c. Consider the amplifier with $2\alpha_{m0}(z) = 0.1\ \text{cm}^{-1}$ and $\alpha_0 = 0.01\ \text{cm}^{-1}$. Find the output power with an input of 1 mW. (5 pts)

4. For a simple resonator with mirrors of curvatures R_1 and R_2 separated by distance d,

a. Write down the ABCD matrix for paraxial ray propagating through one round trip in the cavity (5 pts)

b. Derive the stability (confinement) condition $0 \le g_1 g_2 \le 1$ where $g_i = 1 + d/R_i$. (5 pts)

Part III: 50 points, please describe your answers as complete as possible

1. The concentration of charge carriers in a sample of intrinsic Si is $n_i = 1.5\times10^{10}$ cm^{-3}, recombination lifetime τ = 10 μs, the electronic mobility μ_n = 1450 cm^2/V-s and the hole mobility μ_p = 505 cm^2/V-s. If the material is illuminated with light, and an optical power density of 1 mW/cm^3 at λ_0 = 1 μm is absorbed by the material, determine the percentage increase in its conductivity. The quantum efficiency is η = 0.5. (15 pts)

2. (a) Please draw the luminescence intensity of the LEDs as a function of the material energy bandgap. (10 pts)

(b) According to the (a) results, please proof its peak value at a frequency v_p determined by (10 pts)

$$hv_p = E_g + \frac{1}{2}kT .$$

(c) Show that the full width at a half-maximum (FWHM) of the special intensity is

$\Delta\nu \approx \dfrac{1.8kT}{h}$. (10 pts)

3. Please explain what is the semiconductor? (5 pts)