

Part 1: 50 points, please describe your answers as complete as possible.

1. (15%) An EM wave with an electric amplitude E_i is passing through from medium 1 (refractive index n_1) to medium 2 (refractive index n_2), which reflection and transmission are satisfied with the following equations.

$$r_{\parallel} = \left(\frac{E_r}{E_i} \right)_{\parallel} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$$t_{\parallel} = \left(\frac{E_t}{E_i} \right)_{\parallel} = \frac{2n_1 \cos \theta_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$$r_{\perp} = \left(\frac{E_r}{E_i} \right)_{\perp} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

$$t_{\perp} = \left(\frac{E_t}{E_i} \right)_{\perp} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

where E_r and E_t are amplitudes for reflection and transmission; the symbols of \parallel and \perp individually mean the electric field vectors parallel and perpendicular to the incident plane; θ_1 and θ_2 are incident and refractive angles, respectively. Please illustrate what is the Brewster angle and show the equation relative to the incident angle.

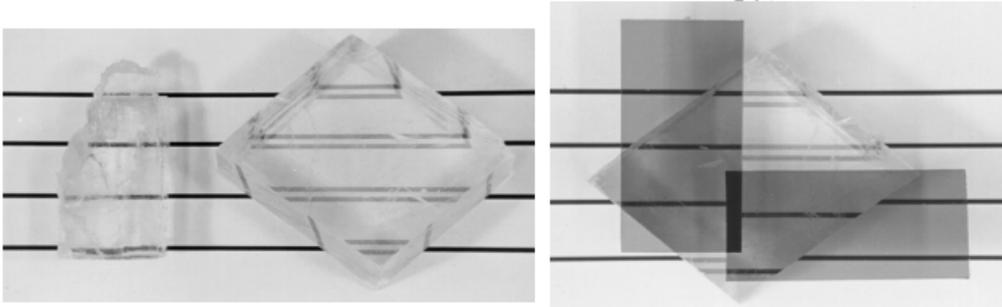
2. (15%) Consider a spherical mirror with a radius of curvature R whose reflectivity varies as

$$\rho(r) = \rho_0 \exp(-r^2/a^2)$$

where r is the radial distance from the center. Show that the (A, B, C, D) matrix of this mirror is given by

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -\frac{2}{2R} - i \frac{\lambda}{\pi a^2} & 1 \end{vmatrix}$$

3. (10%) Please mathematically describe what is the “Airy rings”, and simultaneously illustrate what is the “diffraction-limitation”?
4. (10%) According to the following two images, please make detailed illustration and comparisons to describe what optical phenomena exist in the images? (hint: optical characteristics of isotropic and anisotropic materials)



Part II . Laser Fundamentals and Applications

1. Explain the terminologies and phenomena below:
 - (1) Second-Harmonic Generation (SHG) (4pts)
 - (2) What is Franz-Keldysh effect? Please draw the influence of this effect on the band-gap and absorption spectrum (10pts)
 - (3) Electroluminescence (4pts)
 - (4) Stoke-shift (4pts)

2. (1) Please write down the rate equation for a two level laser system and explain why a two level system is not feasible. (6pts)
(2) Lasers are commonly classified into “three-level” or “four-level” lasers. Please indicate the classification of a Ruby Laser and describe the working principles, such as the characteristics, the energy level diagram, and the pumping, of a Ruby Laser. (10 pts)

3. (1) What voltage shall be applied to accelerate an electron from zero velocity in order that it acquire the same energy as a photon of wavelength equal to 0.87 μm . (4 pts)
(2) Compare the total momentum of the photons in a 10-J laser pulse with that of a 1-g mass moving at a velocity of 1 cm/s and with an electron moving at a velocity of $C_0/10$. (8 pts) (where C_0 is the speed of light, $M_e = 9.1 \times 10^{-31}$ kg)

Part III. Optoelectronic Devices

1. Consider an AlGaInP/InGaP heterostructure laser diode which has an optical cavity of length $300\ \mu\text{m}$. the emission peak is 700nm and the refractive index of InGaP is around 3.5.
- (a) What is the mode integer m of the emission peak and the separation between the modes of the cavity? (5 points) (b) If the optical gain vs. wavelength characteristics has a FWHM wavelength width of about $6\ \text{nm}$ how many modes are there within this bandwidth? (5 points) (c) How many modes are there if the cavity length is $50\ \mu\text{m}$? (5 points)
2. (a) If a p-i-n photodiode has a constant external quantum efficiency of 60% at $200\ \text{nm}$ and $400\ \text{nm}$. What are the theoretical responsivities? (5 points) (b) For a practical GaN p-i-n photodiode, the measured responsivity is far less than the theoretical value when the wavelength of incident light is at $200\ \text{nm}$ or $400\ \text{nm}$. Please explain about it and depict the relation of responsivity (R) vs. wavelength (λ) for an ideal photodiode with 100% quantum efficiency and for a practical GaN p-i-n photodiode. (15 points) (Note: The bandgap of GaN is around $3.4\ \text{eV}$ at 300K)
3. If the width of the relative light intensity vs. photon energy spectrum of an LED is typically around $\sim 3k_B T$, what is the theoretical linewidth, $\Delta\lambda_{1/2}$, in the output spectrum in terms of wavelength?
- (a) InGaN LED emits at a peak wavelength of $470\ \text{nm}$ (5 points)
- (b) AlInGaP LED emits at a peak wavelength of $630\ \text{nm}$. (5 points)
- note: $\frac{\Delta\lambda}{\Delta E_{ph}} \approx \left| \frac{d\lambda}{dE_{ph}} \right|$
4. What is the photoconductive gain for a semiconductor photodetector (5%)