1. (a) A uniform plane wave propagating in the *z* direction is composed of two components, $\vec{E}_x$ and $\vec{E}_y$.

Please write out what are the required conditions of $\vec{E}_x$ and $\vec{E}_y$ in order that the uniform plane wave is linear polarization, circular polarization (RHCP and LHCP), and elliptical polarization, respectively? (10 points)

(b) A circularly polarized wave results from the superposition of two waves that are (i) of the same frequency and amplitude, (ii) plane-polarized in perpendicular directions, and (iii) 90° out of phase. Show that the average value of the Poynting vector for such a wave is the sum of the average values of the Poynting vectors for the two plane-polarized waves. (10 points)

2. A 53W-m^{-2} uniform plane wave in air is obliquely incident on a perfect conductor boundary located at the z=0 plane. The electric field of the incident wave is given by

$$\vec{E}_i(x,z) = \hat{y}E_0 e^{-j4.8\pi x} e^{-j6.4\pi z}$$

(a) Find frequency, and incident angle θ_i. (5 points)

(b) Find the electric field of the reflected wave $\vec{E}_r$. (5 points)

(c) Find the total electric field $\vec{E}_l$ and the nearest positions (with respect to the conductor surface) of its minima and maxima. (10 points)

3. We wish to design an anti-reflective coating for a glass surface in an optical device at a free-space of λ_0 = 600nm. The refractive index of the glass is n_{glass}=1.9.

(a) Determine the refractive index and minimum thickness for the coating. (5 points)
(b) What percentage of the incident power is reflected from the coated glass if the wavelength is changed to 500nm? (5 points)

1. (20pts) A metallic parallel-plate waveguide has plate separation d=1cm, and is filled with teflon, having dielectric constant ϵ'_R=2.1. (a) Determine the maximum operating frequency such that only the TEM mode will propagate. Also find the range of frequencies over which the TE_1 and TM_1 (m=1) modes will propagate, and no higher order modes. [10pts] (b) If the operating wavelength is λ=2mm. How many waveguide modes will propagate? [5pts] (c) Suppose the operating frequency is 25GHz. Consequently, modes for which m=1 and m=2 will be above cutoff. Determine the *group delay difference* between these two modes over a distance of 1cm. This is the difference propagation times between the two modes when energy in each propagates over the 1cm distance. [5pts]

2. (10pts) Two rectangular waveguides are joined end-to-end. The waveguides have identical dimensions, where $a=2b$. One waveguide is air-filled; the other is filled with a lossless dielectric characterized by ϵ'_R. (a) Determine the maximum allowable value of ϵ'_R such that single-mode operation can be simultaneously ensured in *both* waveguides at some frequency. [5pts] (b) Write an expression for the frequency range over which single-mode operation will occur in both waveguides; your answer should be in terms of ϵ'_R, waveguide dimensions as needed, and other known constants. [5pts]

3. (5pts) A *0.5mm* thick slab of glass (n_1=1.45) is surrounded by air (n_2=1). The slab guides infrared light at wavelength $\lambda=1.0\mu$m. How many TE and TM modes will propagate?

4. (10pts) An asymmetric slab waveguide is shown in Fig.A. In this case, the regions above and below the slab have unequal refractive indices, where $n_1<n_3<n_2$. (a) Write, in terms of the appropriate indices, an expression for minimum possible wave angle, θ_1, that a guided mode may have. [5pts] (b) Write an expression for the maximum phase velocity a guided mode may have in this structure, using given or known parameters. [5pts]

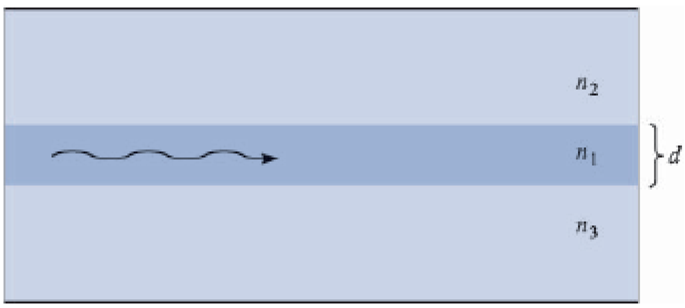

Fig.A

5. (5pts) A step index optical fiber is known to be single mode at wavelengths $\lambda>1.2\mu$m. Another fiber is to be fabricated from the same materials, but it is to be single mode at wavelengths $\lambda>0.63\mu$m. By what percentage must the core radius of the new fiber differ from the old one, and should it be larger or smaller?

1. (15 points) Consider in dielectric material with the electron charge q_e, mass m_e and density N_e. Assume the electron is elastically bound to its nucleus with the elastic constant κ and the effective collision frequency of electrons in material is γ. Please derive the effective permittivity of the material $\varepsilon_{eff}(\omega)$ irradiated by EM wave (Lorentz model).

2. (20 points) Consider a Hertzian dipole with the current ***I*** confined to a wire whose cross-sectional area is small and length is **d*l*** (**d*l*** is also small) (oriented in the z direction). Please derive the electric and magnetic fields in the (a) near-field zone and (b) far-field zone. Hint: Using vector potential **A**, $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$ and

Spherical coordinates:

$$\text{curl}\,\mathbf{A} = \nabla\times\mathbf{A} = \hat{\mathbf{r}}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

3. (15 points) A 2-m long dipole antenna (Hertzian dipole) is excited by a sinusoidal current of amplitude 10 A and frequency 1 MHz. Find the time-average power density radiated by the dipole at a distance of 5 km in a direction which is 0^{o}, 45^{o} and 90^{o} from the axis of the dipole. Hint:

The time-average Poynting vector $\mathbf{S}_{av}$ is simply related to the radiation intensity and is thus given by

$$\boxed{\mathbf{S}_{av}(\theta,\phi) = \hat{\mathbf{r}}\frac{U(\theta,\phi)}{r^2} = \hat{\mathbf{r}}\frac{g_d(\theta,\phi)P_{rad}}{4\pi r^2}} \qquad [6.80]$$

For the Hertzian dipole, the far-zone fields are azimuthally symmetric (i.e., they do not depend on ϕ), so the directive gain is a function only of θ and is given by

$$U_{av} = \frac{P_{rad}}{4\pi} \simeq \frac{(Idl)^2\beta^2\eta}{48\pi^2} \quad \text{watts/steradian}$$

The directive gain is then

$$g_d(\theta) = \frac{U(\theta,\phi)}{U_{av}} \simeq \frac{\frac{1}{2}\left(\frac{Idl\beta}{4\pi}\right)^2\eta\sin^2\theta}{\frac{(Idl)^2\beta^2\eta}{48\pi^2}} = \frac{3}{2}\sin^2\theta \qquad [6.81]$$