Part I: 50 points, Please describe your answers as complete as possible

1. The Glan prism: Consider a light beam incident on a plane boundary from the inside of a calcite crystal (n_o = 1.658, n_e = 1.486). Suppose that the *c* axis of the crystal is normal to the plane of incidence.

(a) (5%) Find the range of the apex angle α such that the ordinary wave is totally reflected. The transmitted wave is thus completely polarized.

(b) (5%) Please identify the polarization direction of the ordinary wave and the extraordinary wave, respectively.

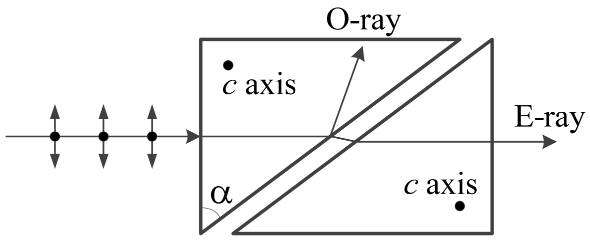


Fig. 1

2. Consider a thick lens with radii of curvature R_1 and R_2 (R_1, R_2 > 0) on its entrance and exit surfaces, an index of refraction *n*, and a thickness *d*.

(a) (7%) Obtain the ABCD matrix of the lens.

(b) (8%) What is its focal distance for light incident from the left?

3. Consider a symmetric Fabry-Perot etalon with mirror reflectivity *R*.

(a) (7%) Derive an analytic expression for the phase delay of the transmitted beam (phase of E_t/E_i)

(b) (8%) Derive an expression for the group delay (derivative of phase with respect to angular frequency). Find the maximum and minimum of the group delay, in units of the cavity transit time. (Assume *n* = 1)

4. (10%) Consider the transmission of 10 Gb/s signals at λ = 1500 nm in a single-mode fiber, with a group velocity dispersion of *D* = 17 ps/nm-km. Determine the pulse spreading after a transmission of distance 100 km.

Part II. (50 points)

5. (10 %) (Photon and Atom)
   Please plot the radiation processes of the spontaneous emission, stimulated emission, and absorption. And explain how to determine their lifetime or decay rate.

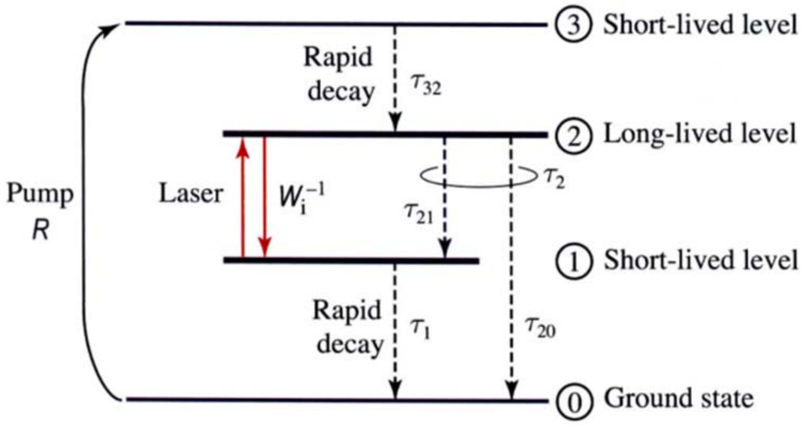
6. (10 %) (Gas Laser)

A CO_2 laser (λ = 10.6 μm) has a resonator of length 100 cm. The refractive index is n=1.

(2-a) (5 %) Determine the mode spacing between the resonator modes.

(2-b) (5 %) The Doppler-Broadening lineshape function can be presented as the Gaussian lineshape $\bar{g}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_D} exp\left[-\frac{(\nu-\nu_0)^2}{2\sigma_D^2}\right]$, where $\sigma_D = \frac{1}{\lambda}\sqrt{\frac{kT}{M}}$ and M is the atomic mass. Please derive the full-width half-maximum (FWHM) Doppler linewidth $\Delta\nu_D$.

7. (10 %) (Lasers)



(3-a) (5 %) Please write down the rate equations of level 1 and level 2.

(3-b) (5 %) Please plot the three-level system.

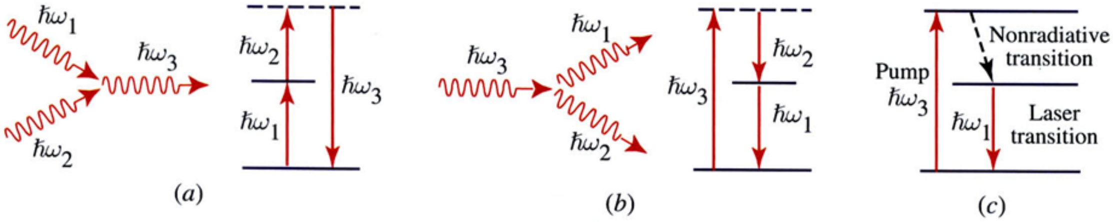
8. (10 %) Intensity Modulators

(4-a) (5 %) An anisotropic medium has two linearly polarized normal modes that propagate with difference, say $\frac{c_0}{n_1}$ and $\frac{c_0}{n_2}$. If the medium exhibits the Pockels effect, then in the presence of a steady electric field E, the two refractive indexed are modified as $n_1(E) \approx n_1 - \frac{1}{2}\mathfrak{r}_1 n_1^3 E$ and $n_2(E) \approx n_2 - \frac{1}{2}\mathfrak{r}_2 n_2^3 E$, where $\mathfrak{r}_1$ and $\mathfrak{r}_2$ are the appropriate Pockels coefficients. After propagation a distance L, the two modes undergo a relative phase retardation given by $\Gamma = k_0[n_1(E) - n_2(E)]L$. If E is obtained by applying a voltage be V between two surfaces of the medium that are separated by a distance d, please derive the "Retardation Half-Wave Voltage".

(4-b) (5 %) Please explain two intensity modulators using an electro-optic modulator.

9. (Second-Order Nonlinear Optics)

(5-a) (5 %) Please explain the physical meaning of Figures (a) (b), and (c).



(5-b) (5 %) (Electro-optic Effect) We examine the optical properties of a nonlinear medium in which nonlinearities of order higher than the second are negligible, so that $\mathcal{P}_{NL} = 2d\mathcal{E}^2$. We consider an electric field $\mathcal{E}(t)$ comprising a harmonic component at an optical frequency ω together with a steady component (at $\omega = 0$), $\mathcal{E}(t) = E(0) + \mathrm{Re}\{\mathrm{E}(\omega)\exp(j\omega t)\}$. Please show the components ($\mathcal{P}_{NL}(0)$, $P_{NL}(\omega)$, and $\mathcal{P}_{NL}(2\omega)$) of nonlinear polarizations, and derive $\Delta n = \frac{2d}{n\epsilon_0}\mathrm{E}(0)$.

Part III (50 points)

10. Derive the minimum detectable optical power for a photodiode operated in the heterodyne mode. (10points)
11. Calculate the smallest temperature increment that can be measured by an infrared detector "looking " at an object at T = 350 K with a background temperature of T = 300 K. The detector has a D_A^* =10^{11} cm(Hz)$^{1/2}$/W and responds to $\Delta\lambda$ ~0.1λ centered on λ = 10 μm. The output circuit bandwidth is $\Delta f = 10^3$ Hz. (10 points)
12. Solve for the carrier density modulation $\mathrm{N} = N_0 + N_1 e^{i\omega_m t}$ in a semiconductor laser whose current is modulated at

    $$\mathrm{I} = I_0 + I_1 e^{i\omega_m t}$$

    $$\omega_m = \text{ Modulation frequency } \ll \omega_{opt}$$

    (15 points)
13. Assume $\varepsilon = \varepsilon_0 - aN$, where a is a constant, and that the instantaneous frequency of the semiconductor laser obeys

    $$\frac{\Delta\nu}{\nu} = -\frac{\Delta\varepsilon}{\varepsilon}$$

    Find the form of the laser optical field due to the current modulation. (10 points) What is the (phase) modulation index of the field? (5 points)