Part I: 50 points, please describe your answers as complete as possible

1.) **E** and **B** can be determined from the potentials V and **A**, which are related by the Lorentz condition, in the time-harmonic case. The vector potential **A** was introduced through the relation $\vec{B} = \nabla \times \vec{A}$ because of the solenoidal nature of **B**. In a source free region, $\nabla \bullet \vec{E} = 0$, we can define another type of vector potential $\mathbf{A_e}$, such that $\vec{E} = \nabla \times \vec{A}_e$. Assuming harmonic time dependence:

(a) Express **H** in terms of $\mathbf{A_e}$.(8%)

(b) Show that $\mathbf{A_e}$ is a solution of a homogeneous Helmholtz's equation. (7pts)

Lorentz condition: $\nabla \bullet \vec{A} + j\omega\mu\varepsilon V = 0$

2.) Given that $\vec{E} = \vec{a}_y 0.1 \sin(10\pi x)\cos(6\pi 10^9 t - \beta z)$ in air. Find **H** and β. (10pts)

3.) A uniform plane wave with $\vec{E}_i(z,t) = \vec{a}_x E_{i0} \cos\omega(t - \frac{z}{u_p})$ in medium 1 (ε_1, μ_1) is incident normally onto a lossless dielectric slab (ε_2, μ_2) of a thickness *d* backed by a perfectly conducting plane, as shown in Fig. 1. Please find

(a) $\mathbf{E}_r$(z, t) (electrical field vector of reflection wave in midum 1), (b) $\mathbf{E}_1$(z, t)(electrical field vector in medium 1), (c) $\mathbf{E}_2$(z, t) (electrical field vector in medium 2), (d) time average Poynting vector or average power density in medium 1 $(\mathbf{P}_{av})_1$, (e) time average Poynting vector or average power density in medium 2 $(\mathbf{P}_{av})_2$ (f) determine the thickness *d* that makes $\mathbf{E}_1$(z, t) the same as if the dielectric slab were absent. (15pts)

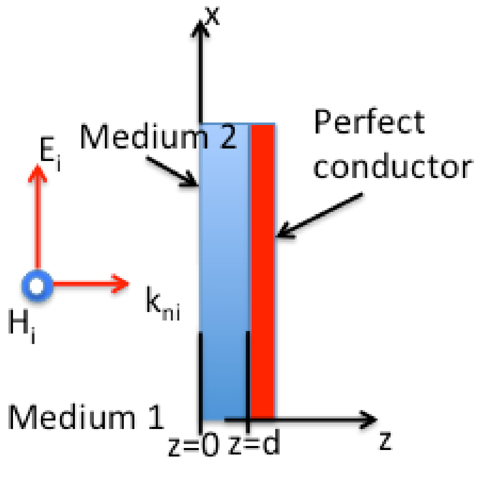

Fig. 1

4.) A 10(kHz) parallel polarized electromagnetic wave in air is incident obliquely on an ocean surface at near grazing angle $\theta_i = 88^o$. Using $\varepsilon_r = 81$ and $\mu_r = 1$ and $\sigma = 4$(S/m) for sea water, find (a) the angle of refraction θ_t, (b) the transmission coefficient, $\tau_{||}$, (c) the ratio of the transmitted time average Poynting vector and incident time average Poynting vector $(\mathbf{P}_{av})_t/(\mathbf{P}_{av})_i$, and (d) the distance below the ocean surface where the field intensity has been diminished by 30 dB. (10pts)

($\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = (1/36\pi) \times 10^{-9}$ F/m$= 8.854 \times 10^{-12}$ F/m, $c = 3 \times 10^8$ m/s)

Part II: 50 points, please describe your answers as complete as possible

1. For an infinite dielectric (made of simple medium) slab waveguide of thickness *d* situated in air, answer the following questions about the instantaneous expressions of all nonzero field components, eigenvalue equation, and cutoff frequency for odd TE modes. Remember to show your detailed steps (30pts)
   (a) In a source-free environment, rewrite the real-field Maxwell's equations into complex-field Maxwell's equations for a time-harmonic electromagnetic wave.
   (b) Derive the inter-relationships among all complex-field components according to (a).
   (c) Derive a wave equation for the complex magnetic field based upon (b).
   (d) Assume the electromagnetic wave propagates along the z-direction in Cartesian coordinates. Based on (c), derive the z-components of the complex magnetic field in all regions for odd TE modes, where the boundary condition, the tangential component of the magnetic field must be continuous across the boundary for non-conducting medium, is used.
   (e) Based on (b) and (d), derive other complex-field components in all regions for odd TE modes.
   (f) Based on (e), obtain the instantaneous expressions of all the field components for odd TE modes in all regions.
   (g) Based on (e), derive the eigenvalue equation for odd TE modes by using the boundary condition, the tangential component of the electric field must be continuous across the boundary.
   (h) Based on (g), derive the cutoff frequency for odd TE modes.

2. An air-filled circular waveguide has an inner diameter of 2.383 cm. (20pts)
   (a) Determine the cutoff frequencies of the TE_{11}, TM_{01}, and TE_{21} modes.
   (b) Find the modes that will propagate through this guide at 10 GHz.
   (c) Find the guide wavelengths for all the propagating modes at 10 GHz.
   (d) Find the frequency range within which only the TE_{11} mode propagates.

Part III: 50 points, please describe your answers as complete as possible

1. (25pts)

The current along an isolated and terminated traveling-wave antenna of length *L* is given as

$$I(z) = I_0 e^{-j\beta z}.$$

(a) Find the far-zone vector potential, **A**(R, θ). (10pts)

(b) Determine **H**(R, θ) and **E**(R, θ) from **A**(R, θ). (10(pts)

(c) Sketch the radiation pattern for *L* = λ/2. (5pts)

2. (10pts)

Consider a rectangular waveguide of dimensions *a*, *b* filled with an isotropic plasma with plasma frequency ω_p.

(a) Find the cutoff frequency for the dominant TE_{10} mode in terms of *a*, *b*, and ω_p.

(b) Determine the quality factor Q if a plasma-filled cubical resonator for the TE_{101} mode.

3. (15pts)

(a) Show that the effective permittivity of an isotropic (unmagnetized) plasma with collisions can be expressed as

$$\varepsilon_{eff}(\omega) = \varepsilon_0 \left[1 - \frac{\omega_p{}^2}{\omega^2 + \nu^2} + j \frac{\nu \omega_p}{\omega(\omega^2 + \nu^2)} \right].$$

where ν is the collision frequency and ω_p is the plasma frequency.

(b) Using this expression to find a simple approximation for the attenuation rate (in dB-m^{-1}) for a 30 MHz wave passing through the lower ionosphere (assume $\nu \cong 10^7$ s^{-1}, $N_e \cong 10^8$ m^{-3}).