Part I (50 points)

1. a) Derive the Maxwell's equations

$$\nabla \times \vec{E} = -j\omega\vec{B}$$
$$\nabla \cdot \vec{D} = \rho$$
$$\nabla \times \vec{H} = \vec{J} + j\omega\vec{D}$$
$$\nabla \cdot \vec{B} = 0$$

from Coulomb's law, Ampere's law, Faraday's law, and the principle of conservation of electric charge. (10%)

b) Explain the physical meanings of Maxwell's equations. (5%)

b) Derive the wave equations of $\vec{E}$ and $\vec{H}$. (5%)

c) Solve these two wave equations to obtain time-harmonic plane waves in a lossless medium. Assume that the waves propagate in the *z*-direction. (5%)

2. a) Prove that the time-average Poynting vector is

$$\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\{\vec{E} \times H^*\}. \quad (10\%)$$

b) A time-harmonic uniform plane wave propagates along +*z* direction in the general case of lossy medium. Its instantaneous electric field is

$$\vec{E}(z,t) = \hat{x}C_1 e^{-\alpha z}\cos(wt - \beta z)$$

where C_1, α, and β are the amplitude, attenuation constant, wave number, respectively. Find the time-average Poynting vector. (5%)

3. Consider two circular polarized waves traveling in the same direction transmitted by two different satellites operating at the same frequency given by

$$\vec{E}_1 = E_{01}(\hat{x} + \hat{z}e^{j\pi/2})e^{j\beta y}$$

$$\vec{E}_2 = E_{02}(\hat{x} + \hat{z}e^{j3\pi/2})e^{j\beta y}$$

where E_{01} and E_{02} are real constants. a) If the time-average power densities of these two waves are equal, find the polarization of the total wave. (5%) b) Repeat part (a) for the case when the time-average power of the first wave is 4 times the time-average power of the second wave. (5%)

Part II (50 points)

1. A uniform plane wave with perpendicular polarization is obliquely incident on the interface from dielectric media 1(ε_1 and μ_1) to medium 2 (ε_2 and μ_2). The angles of incidence and transmission are θ_i and θ_t, respectively.
   (a) Please find the reflection coefficient and transmission coefficient. (10 pts)
   (b) Assuming $\varepsilon_2 < \varepsilon_1$ and both medium being nonmagnetic, find critical angle and verify that the average power transmitting into medium 2 vanished. (8 pts)
   (c) For glass isosceles triangular prisms with index of 2, as shown in Fig. 1, please calculate the percentage of the incident light power reflected back by the prism. Does this percentage depend on the polarization? (7 pts)

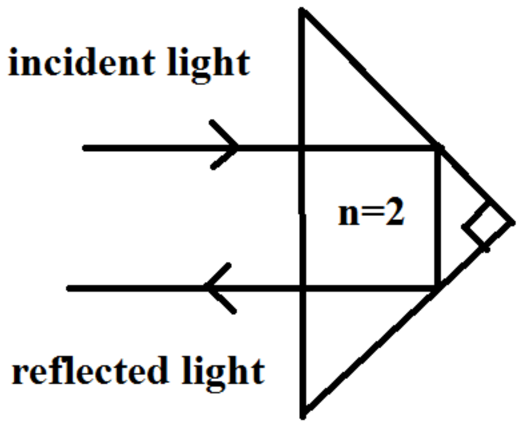


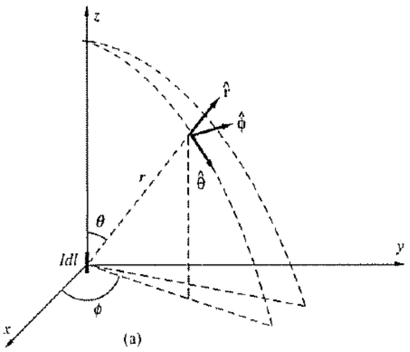
Fig. 1

2. In a parallel-plate waveguide, the plates, located at y=0 and y=a, are assumed to be infinite in extent in the x and z directions. Let us suppose that TM waves propagate in the +z direction.
   (a) Drive the instantaneous field expression for TM_1 mode from the homogeneous vector Helmholtz's equations. (10 pts)
   (b) Calculate the group velocity of TM_1 mode. (5 pts)
   (c) The energy-transport velocity is defined as the ratio of the time-average propagated power to the time average stored energy per unit guide length. Determine the energy-transport velocity of TM_1 mode and verify that the energy-transport velocity is equal to the group velocity. (10 pts)

Part III (50 points)

1. (15%) (a) Write down the ***time varying*** *Maxwell equations* in differential form with current **J** and charge density ρ.(b) from (a) derive the non-homogeneous *wave equation* for vector potential **A** and scalar potential V. Specify the *gauge* you used. (C) Write down the solutions of **A** and V in integral from.
2. Hertz antenna (10%)

For a Hertzian dipole with length of ***dl***, driven by current ***I*** in the *z* direction.



(a)

(a) From problem 1(c) in Part III, <u>derive</u> the ***H*** fields from **A** and derive ***E*** from ***H*** as the followings:

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}\right]e^{-j\beta r} \qquad E_r = -\frac{Idl}{4\pi}\eta\beta^2 2\cos\theta\left[\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta\beta^2 \sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r} \; E_\phi = 0 \quad \text{where} \quad \beta = \omega\sqrt{\varepsilon_0\mu_0} \quad \text{and} \quad \eta = \sqrt{\mu_0/\varepsilon_0}$$

Hint: using unit vector $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$ and

$$\nabla\times A = \frac{\hat{r}}{r\sin\theta}\left(\frac{\partial(A_\phi \sin\theta)}{\partial\theta}-\frac{\partial A_\theta}{\partial\phi}\right)+\frac{\hat{\theta}}{r\sin\theta}\left(\frac{\partial A_r}{\partial\phi}-\sin\theta\frac{\partial(rA_\phi)}{\partial r}\right)+\frac{\hat{\phi}}{r}\left(\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta}\right)$$

(b) Identify the far-zone field (hint only two components)

3. Rectangular Waveguide (25%)

For a rectangular Perfect conducting waveguide, assume the wave is guiding in the z direction with z dependence as $e^{-\gamma z}$.

(a) Write down the *Maxwell equations* for all the six field components E_x , E_y , E_z , H_x , H_y , H_z in terms of γ , $j\omega$, μ , ε (replace $\partial/\partial z \to -\gamma$, $\partial/\partial t \to j\omega$) (first order differential form)

(b) write down the wave equations for E_z and H_z (second order differential form)

(c) Find E_x , E_y , H_x , H_y in terms of only E_z and H_z with parameter $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

(d) for the Transverse Electric (**TM**) wave, H_z=0, what are the boundary conditions and solve for E_z and write down other components E_x , E_y , H_x , H_y

(e) what are the cutoff frequency and phase velocity for TM_{mn} modes