Part I (50%)

1. A linearly chirped Gaussian pulse can be expressed as

$$E(t) = E_0 \exp[-\alpha t^2]\exp[i(\omega_0 t + \beta t^2)]\,.$$

   (a) Show the instantaneous frequency (3%)
   (b) Drive the frequency-domain field E(ω).

   (hint: $\exp[-ct^2] = \int_{-\infty}^{\infty} \sqrt{\frac{1}{4\pi c}} e^{-\Omega^2/4c} e^{i\Omega t} d\Omega$, c is a constant) (7%)

   (c) Show the group delay, where its definition in frequency domain is analogous to that of the instantaneous frequency in time domain. (3%)

2. A Gaussian beam with the wavelength of λ propagates in the +z direction. Assume that w(0) is the beam spot size and R(0) < 0 is the radius of curvature at z = 0.
   (a) What is the complex beam radius q(0) at z = 0 and the ABCD law? (4%)
   (b) Please find the size of the beam waist and the position of the beam waist. (7%)
   (c) Please find the position z_m which has the maximal curvature based on the ABCD law. (6%)

3. Polarization optics
   (a) What is the index ellipsoid? (2%) Considering the case of a uniaxial crystal with z axis being the optic axis, please show the figure and the formula for the intersection of the index ellipsoid with the y-z plane. (The principal indices of the refraction n_o and n_e must be labelled in the figure.) (4%)
   (b) When a wave propagates in y-z plane and an angle θ to the z (optic) axis. Drive the index of the extraordinary wave $n_e(\theta)$ as a function of θ, n_o, and n_e. (7%)
   (c) Please use one wave plate to transform the wave from x-direction linear polarization to right-hand circular polarization. (The kind and the fast axis of the wave plate must be identified.) Explain this setup by Jones matrix method. (7%)

Part II. (50 %)

1. (10 %) (Lasers)
Semiconductor lasers are often fabricated from crystals whose surfaces are cleaved along crystal planes. These surfaces act as reflectors by virtue of Fresnel reflection, and therefore serve as the mirrors of a Fabry-Perot cavity. An expression for the intensity reflectance is provided as $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$.
(1-A) (5 %) Consider a crystal placed in air (n=1) whose refractive index n=3.6 and loss coefficient α_s=0.5 cm^{-1}. The light reflects between two parallel surfaces separated by a distance d=0.5 mm. Determine (a) the spacing between resonance frequencies v_F, (b) the overall distributed loss coefficient α_γ, (c) the finesse F, (d) the spectral width of a mode δv.
(1-B) (5 %) If the free-space wavelength of the generated light $\lambda_0 = 1.5\ \mu m$, estimate the longitudinal mode number q.

2. (10 %) (Resonator optics)
Determine the frequency spacing, and spectral width, of the modes of a Fabry-Perot resonator whose mirrors have intensity reflectances of 0.98 and 0.99 and are separated by a distance d=100 cm. Assume that the medium has refractive index n=1 and negligible losses.

3. (10 %) (Pumping Schemes)
Please plot the pumping schemes of 4-level pumping and 3-level pumping. Must plot all possible transitions and indicate the energy level is short-lived level or long-lived level.

4. (10 %) (Laser Oscillation)
(4-A) (5 %) Please use Figure 1 to explain the laser oscillation. (please indicate the operation regions, like (a) Gain>Loss, and so on.)

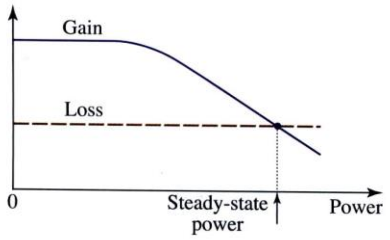

Fig. 1

(4-B) (5 %) Use the phase-shift condition $\left(\nu + \frac{c}{2\pi}\frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu) = \nu_q\right)$ to plot the frequency spectrum and explain the "frequency pulling".

5. (10 %) (Electro-Optic Effect in $LiNbO_3$)

The $LiNbO_3$ is a uniaxial crystal with ordinary and extraordinary refractive indexes (n_0 and n_e). The applied electric field is parallel to the z axis. The electro-optic tensor of $LiNbO_3$ in the form of

$$r_{ij} = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}.$$

(5-a) (5 %) Please derive the birefringence of $n_z - n_y$ when the propagation of light along the x axis.

(5-b) (5 %) After propagation a distance L, the two modes undergo a relative phase retardation given by

$$\Gamma = k_0[n_z - n_y]L = k_0\left[(n_e - n_o) - \frac{1}{2}(n_e^3 r_{33} - n_o^3 r_{13})E(n_1 - n_2)\right]L.$$

If E is obtained by applying a voltage be V between two surfaces of the medium that are separated by a distance d, please derive the “Retardation Half-Wave Voltage”.

Part III. (50%)

1. The gain coefficient of the InGaAsP semiconductor optical amplifier as a function of the photon energy with the injected-carrier concentration (Δn) as a parameter (shows in the Fig. 2). Please explain that both the amplifier bandwidth and the peak value of the gain coefficient increase with the injected carrier concentration. (20%)

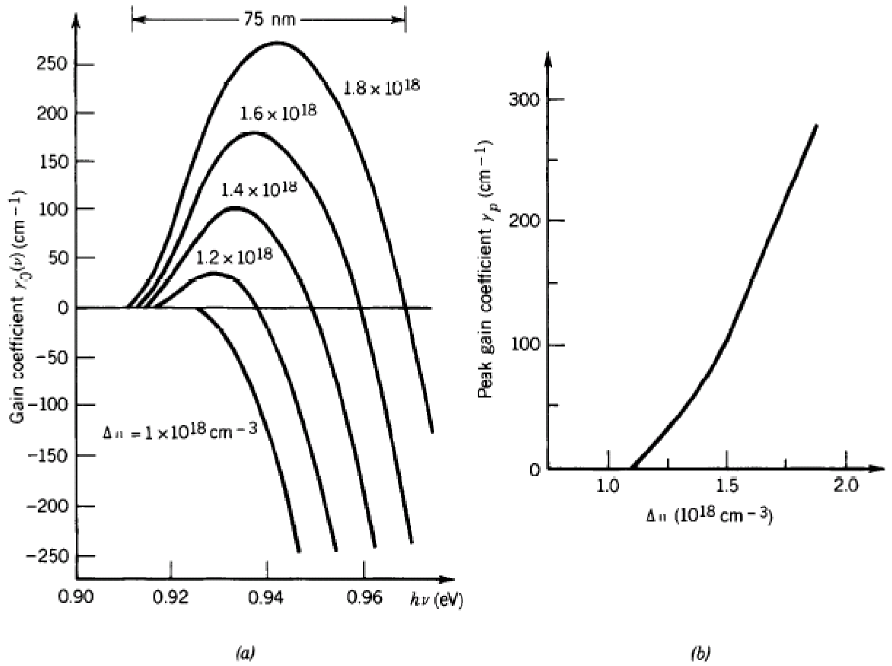

Fig. 2

2. Please draw the luminescence intensity of the LEDs as a function of the material energy bandgap. (10%)

3. (a)What is the semiconductor? (5%)
   (b)What is the degenerate semiconductor? (5%)
   (c)What is the operated different between photodetectors and solar cells? (5%)
   (d) What is the different between lasers and LEDs? (5%)