Part I: 50%

1. **Maxwell's equations: (10 points)**
   Please show that if $\left(\vec{E},\vec{H}\right)$ are the solutions of source-free Maxwell's equations in a simple medium characterized by ε and μ, then $\left(\vec{E}',\vec{H}'\right)$ are also the solutions of source-free Maxwell's equations when the following relations are satisfied.

$$\vec{E}' = \eta\vec{H}$$

$$\vec{H}' = -\frac{\vec{E}}{\eta}$$

   In the above relations, $\eta = \sqrt{\mu/\varepsilon}$ is called the intrinsic impedance of the medium.

2. **Electromagnetic wave with boundary conditions (15 points)**
   An uniform EM plane wave propagating in air with electric field of $\mathbf{E}_i(z) = \mathbf{a}_x E_0 \exp(-j\beta_0 z)$ normally impinges onto the material surface at z=0, which is a highly conducting medium with constitutive parameters ε_0, μ ,η_c and σ (σ/ωε_0>>1). In addition, $\eta_c = \sqrt{j\omega\mu/\sigma}$ and $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\pi$ are the intrinsic impedance of the air and the conducting medium, respectively.

(a) Find the reflection coefficient. (5 points)
(b) Derive the expression for the fraction of the incident power absorbed by the conducting medium. (10 points)

3. **Polarization of electromagnetic waves: (15 points)**

(a) A uniform plane wave propagating in z-axial direction is composed of two components, $\vec{E}_x$ and $\vec{E}_y$. Please write out what are the required conditions of $\vec{E}_x$ and $\vec{E}_y$ in order that the uniform plane wave is linear polarization, circular polarization (RHCP and LHCP), and elliptical polarization, respectively? (5 points)
(b) A circularly polarized wave results from the superposition of two waves that are (i) of the same frequency and amplitude, (ii) plane-polarized in perpendicular directions, and (iii) 90° out of phase . Show that the average value of the Poynting vector for such a wave is the sum of the average values of the Poynting vectors for the two plane-polarized waves. (10 points)

4. **Dispersion in sea water: (10 points)**
   A uniform plane electromagnetic wave in free space propagates with the speed of light, namely, $c \simeq 3\times10^8$m-s^{-1}. In a conducting medium, however, the velocity of propagation of a uniform plane wave depends on the signal frequency, leading to the "dispersion" of a signal consisting of a band of frequencies. For sea water ($\sigma = 4$ S-m^{-1}, $\varepsilon_r = 81$, and $\mu_r = 1$), show that for frequencies much less than ~890MHz, the velocity of propagation is approximately given by $v_p \simeq k_1\sqrt{f}$, where k_1 is a constant. What is the value of k_1?

Part II : 50%

1. (11%) In studying the wave behavior in a straight waveguide having a uniform but arbitrary cross section it is expedient to find general formulas expressing the transverse field components in terms of their longitudinal components. We write

   $E = E_T + \vec{a}_z E_z$

   $H = H_T + \vec{a}_z H_z$ where the subscript T denotes "transverse".

   $\nabla = \nabla_T + \vec{a}_z \dfrac{\partial}{\partial z}$

   Prove the following relations for time harmonic excitation:

   $a) E_T = -\dfrac{1}{h^2}(\gamma \nabla_T E_z - \vec{a}_z j\omega\mu \times \nabla_T H_z)$

   $b) H_T = -\dfrac{1}{h^2}(\gamma \nabla_T H_z + \vec{a}_z j\omega\mu \times \nabla_T E_z)$

   where h^2 is $h^2 = \gamma^2 + k^2$

2. (12%) Consider a dielectric slab waveguide with thickness d and refractive indices of 1.5 (for the guide) and 1.48 (for the cladding). Find (a) all the propagating modes at λ = 2 μm if d = 5 μm. (b) Repeat part (a) if d = 15 μm. (C) Repeat part (a) for cladding layer with refractive index of 1.49. (d) Please find the maximum d of the dielectric slab waveguide with indices of 1.55 (for the guide) and 1.5 (for the cladding) to allow only TE_1 mode at λ = 1 μm.

3. (12%) The z component of the magnetic field of the TE_{11} mode traveling in an air filled rectangular waveguide along the z direction is

   $H_z = C\cos 50\pi x \bullet \cos 100\pi y \bullet e^{-j100\pi z}$

   (a) Find the waveguide dimensions and the operating frequency.
   (b) Find the constant C if the time-average power carried by TE_{11} mode in the z direction is 10kW.
   (c) Find all the other nonzero field components of the TE_{11} mode.
   (d) Find the maximum time-average power-carrying capacity of this waveguide, taking the breakdown electric field of air to be 15kV-(cm)$^{-1}$ (with a safety factor of 2 at sea level).

4. (15%) Consider an air-filled circular waveguide system operating in the orthogonally polarized TE_{11} modes. The r component of the total electric field phasor in the waveguide is given by

   $E_r \cong \dfrac{C_1}{r} J_1(80r)(\sin\phi + \cos\phi)e^{j47z}$ where both r and z are in meters. $J_n(r)$ is nth-order Bessel function.

   (a) Find the inner diameter of the waveguide and the operating frequency.
   (b) Verify that the waveguide is indeed operating in the dual polarized dominant modes.
   (C) Find the all other nonzero field components corresponding to the orthogonally polarized TE_{11} modes.

The equations you might need for this problem are

$$\vec{H}_{tr} = \vec{H}_r + \vec{H}_\phi = -\frac{\bar{\gamma}}{\bar{\gamma}^2 + \omega^2 \mu\varepsilon} \nabla_{tr} H_z$$

$$E_{tr} = \vec{E}_r + \vec{E}_\phi = -\frac{j\omega\mu}{\bar{\gamma}} \vec{H}_{tr} \times \hat{z}$$

$$\nabla_{tr} = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi}$$

Parameters: light speed c = 3 x 10^8 m/s, Permittivity of free space ε_0 = 8.854 x 10^{-12} F/m, Permeability of free space μ_0 = 4 π x 10^{-7} H/m

Part III : 50%

1. (10%) For a rectangular cavity with length of (a,b,c) in (+x,+y,+z) direction, please derive the resonant frequency of its TE^{z}_{mnp} modes.

2. (10%) According to (1), please derive the quality factor (Q) for m=1, n=0, p=1 mode.

3. (10%) From (1) and (2), for a square based (a=c) cavity of rectangular cross section is constructed of an X-band (8.2-12.4 GHz) copper ($\sigma = 5.7x10^{7} S/m$) waveguide that has inner dimensions of a=2.286 cm and b=1.1016 cm. For the dominant TE_{101} mode, determine the Q of the cavity. Assume a free-space medium inside the cavity.

4. (10%) Please explain the Hall effect.

5. (10%) For an elementary electric dipole (Hertzian Dipole), the phasor vector potential is expressed as

$$\boldsymbol{A}(\boldsymbol{r}) = \hat{z}\frac{\mu_0 I dl'}{4\pi}\left(\frac{e^{-j\beta r}}{r}\right), \beta = \omega\sqrt{\mu_0\varepsilon_0}$$

Please derive phasor magnetic field **H** and phasor electric field **E.**