博士班 103 學年度第 1 學期資格考 光電子學 科試題 (共 4 頁，第 1 頁)

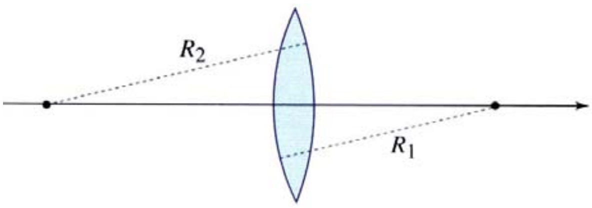
Part I (50 points)

1. Please plot the setup and describe the working principles for the two polarization devices:
   (a) (5 pts) Optical isolator making use of a Faraday rotator, two polarizers, and a mirror.
   (b) (5 pts) Electro-optic modulator (controller of optical intensity) making use of a phase retarder and two polarizers.

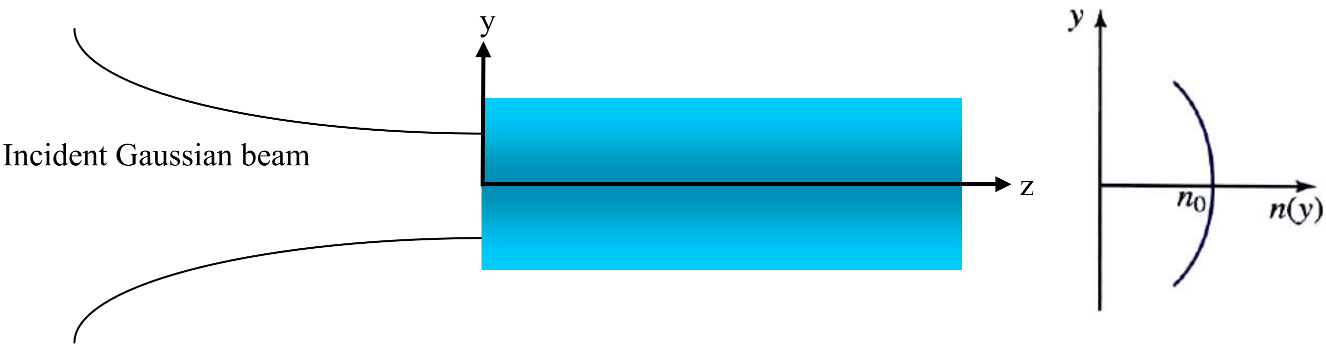
2. (10 pts) Show that the complex amplitude transmittance of the double-convex lens (also called a spherical lens) is given by the following formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

where R_1 and R_2 are the radii of the first and second spherical surfaces, respectively.



3. (15 pts) The ABCD matrix of a SELFOC graded-index slab with quadratic refractive index $n(y) \approx n_0(1-\frac{1}{2}\alpha^2 y^2)$ and length d is A = $\cos\alpha d$, B = $(1/\alpha)\sin\alpha d$, C = $-\alpha\sin\alpha d$, D = $\cos\alpha d$ for paraxial rays along the z direction. A Gaussian beam of wavelength λ_0, waist radius W_0 in free space, and axis in the z direction enters the slab at its waist. Use the ABCD law to determine an expression for the beam width in the y direction as a function of d.



4. (a) (8 pts) In a conducting medium with a free-space-like dielectric property (e.g. ionized gas) with $\varepsilon = \varepsilon_0$ and linear conductive property of **J**=σ**E** (**J**: electric current density, σ: conductivity, **E**: electric field complex-amplitude vector), prove that the associated frequency-dependent effective electric permittivity ε_{eff} is

$$\varepsilon_{eff} = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right),$$

where $\omega_p = (\sigma_0/\varepsilon_o\tau)^{1/2}$ is known as the plasma frequency, τ is a relaxation time with $\omega >> 1/\tau$, σ_0 is the low-frequency conductivity, if the incident wave is a monochromatic wave $\mathcal{E}(t) = \mathrm{Re}\{E\exp(j\omega t)\}$.

(b) (7 pts) An unpolarized plane wave is incident from free space onto a quartz crystal (n_e=1.553 and n_o=1.544) at an angle of incidence 30^o. The optic axis lies in the plane of incidence and is perpendicular to the direction of the incident wave before it enters the crystal. Determine the directions of the wavevectors of the two refracted components.

Part II (50 points)

1. Resonator Optics

(a) Why spherical-mirror resonators are preferred over planar-mirror resonators in most solid state lasers? (5 pts)

(b) Show that the Gaussian beam is a mode of the spherical-mirror resonator. (10 pts)

2. Photons and Atoms, Lasers

(a) Please write down the rate equation for a two level laser system and explain why a two level system is not feasible.(5 pts)

(b) Lasers are commonly classified into “three-level” or “four-level” lasers. Please indicate the classification of an Nd-YAG Laser and the working principles, such as the material, the energy diagram, and the pumping source. (10 pts)

3. Nonlinear Optics

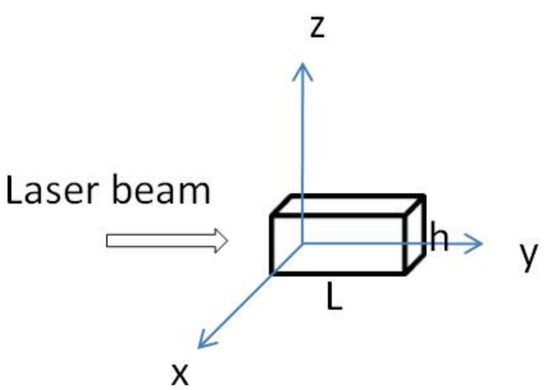
Show that SHG can occur in a third-order nonlinear medium with an applied static electric field. (10 pts)

4. Electro-Optic Modulation of Laser Beams

Lithium Niobate, $LiNbO_3$, is a uniaxial crystal with 3m point symmetry, which has been used frequently in optical communication systems as EO modulators. Its electrooptical tensor has the following form:

$$\begin{bmatrix} 0 & -r22 & r13 \\ 0 & r22 & r13 \\ 0 & 0 & r33 \\ 0 & r51 & 0 \\ r51 & 0 & 0 \\ -r22 & 0 & 0 \end{bmatrix}$$

A laser beam propagating along y axis incidents at the crystal as showing below:

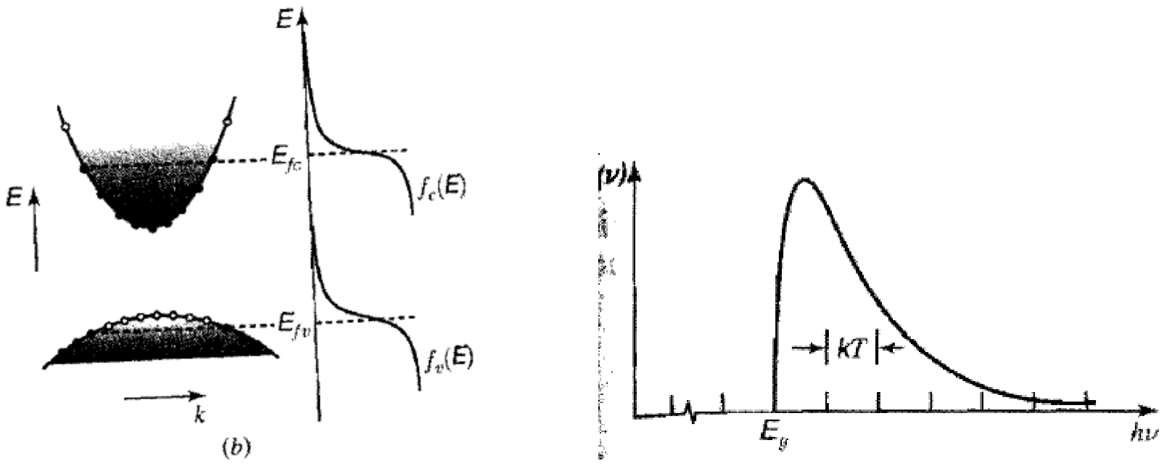


Metallic films are deposited on the top and the bottom face of the crystal to serve as the electrodes to supply RF modulating voltage $V \cdot \sin(\omega t)$ along the z direction. Please determine the resultant changes in refractive indices and the equation of index ellipsoid. (10 pts)

Part III (50 points)

1. (15 pts) photodetectors
   (a) (5 pts) Describe the following two type of photodetectors (i) p-i-n (ii) Avalanche photodiodes
   (b) (5 pts) compare the noise in these two type of photodetectors
   (c) (5 pts) what is the advantage of using the Schottky-Barrier Photodiodes
2. Electroluminescence of semiconductors (20 pts)

   For sufficiently weak injection ($E_c - E_{fc} \gg kT$ and $E_c - E_{fc} \gg kT$), the Fermi functions $f(E) = 1/\left(e^{(E-E_f)/kT} + 1\right)$ can be expressed by its exponential tails, such as $f_c(E) \approx e^{-\frac{E-E_f}{kT}}$ and $1 - f_v(E) \approx e^{-\frac{E_f - E}{kT}}$. The luminescence rate is defined as $r_{sp}(\nu) = \frac{1}{\tau_r}\rho(\nu) f_e(\nu)$, where the optical joint density of states is $\rho(\nu) = \frac{(2m_r)^{3/2}}{\pi\hbar^2}\sqrt{h\nu - E_g}$, and the probability of emission is $f_e(\nu) = f_c(E_2)\left[1 - f_v(E_1)\right]$ (effective mass : $1/m_r = 1/m_c + 1/m_v$, conduction band state of energy $E_2 = E_c + m_r/m_c(h\nu - E_g)$ and valence-band state of Energy $E_1 = E_2 - h\nu$)



   (a) (5 pts) Show that $r_{sp}(\nu) = D\sqrt{h\nu - E_g}\, e^{-\frac{h\nu - E_g}{kT}}$ hν > E_g where $D = \frac{(2m_r)^{3/2}}{\pi\hbar^2\tau_r} e^{\left(\frac{E_{fc} - E_{fv} - E_g}{kT}\right)}$

   (b) (5 pts) Show that the spectral intensity described by (a) has peak value at frequency ν_p by

   $h\nu_p = E_g + \frac{1}{2}kT$

   (c) (5 pts) Show that the full width at half-maximum (FWHM) of the spectral intensity is

   $\Delta\nu \approx 1.8kT/h$

   (use $1 + x - \frac{1}{4}e^x = 0$ has root at 2.6926, and $1 - x - \frac{1}{4}e^{-x} = 0$ has root at 0.8982)

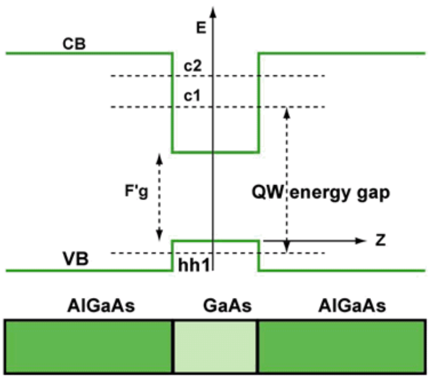
   (d) (5 pts) From (c) Find the corresponding wavelength spread $\Delta\lambda$ in [nm] at T=300^0 K for λ_p=0.8μm (use k=1.38e-23 J/K, h=6.626e-34 J-s)

3. Quantum confinement effect on the density of states (15 pts)

(a) (5 pts) A quantum well structure is a double hetero-structure consisting of an ultrathin layer of semiconductor material with band-gap smaller than that of the surrounding material such as GaAs surrounded by AlGaAs as shown in figure below. The sandwich forms 1D conduction and balance-band rectangular potential wells within which electrons and holes are confined: electrons in the conduction-band well and holes in the valence-band well. A sufficiently deep potential well can be approximated as an infinite rectangular potential well. From 1D Schrodinger equation, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = E\psi$, show that the energy levels E_q of a particle of mass m is $E_q = \frac{\hbar^2}{2m}\left(\frac{q\pi}{d}\right)^2$, where q=1,2,3,…..

(b) (5 pts)From the electron energy-momentum relation $E = E_c + \frac{\hbar^2 k_1^2}{2m_c} + \frac{\hbar^2 k_2^2}{2m_c} + \frac{\hbar^2 k_3^2}{2m_c}$ and the definition of density of state $\rho(k) = \frac{k^2}{\pi^2}$ (3D bulk), $\rho(k) = \frac{k}{\pi d_1}$ (2D well with well thickness d_1), $\rho(k) = \frac{1}{\pi d_1 d_2}$ (1D wire with cross section $d_1 d_2$), and its relation to the density of state in Energy diagram $\rho_c(E)dE = \rho(k)dk$ or $\rho_c(E) = \frac{1}{dE/dk}\rho(k)$, write down the density of states $\rho_c(E)$ for (i) bulk (ii) quantum well (iii) quantum wire (iv) quantum dot semiconductors.

(c) (5 pts) Also schematically illustrate the $\rho_c(E)$ and $\rho_v(E)$ diagram for (i)~(iv) from (b)



A Quantum Well Structure