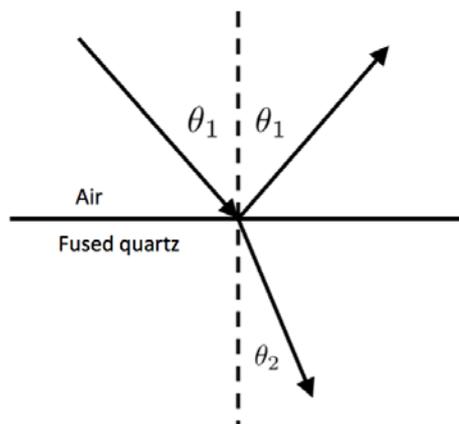


1.  
 (a) (10%) Consider a source-free and lossless medium; derive the wave equation for  $\mathbf{E}$  from Maxwell's equations

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{D} &= 0 \\ \nabla \times \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0\end{aligned}$$

- (b) (5%) Now, for the special case that  $\mathbf{E}$  is independent of  $x$  and  $y$ , write the one-dimensional wave equation for  $E_x$ .  
 (c) (5%) The solution of the wave equation in (b) should represent waves propagating in the  $+z$  and  $-z$  directions. What is  $E_x$ ?  
 (d) (5%) What is the velocity of the propagating wave?

2.  
 Consider a circularly polarized wave, incident at an angle  $\theta_1=60^\circ$  from air onto fused quartz with  $\epsilon_r=3.78$ .  
 (a) (5%) Find  $\theta_2$   
 (b) (15%) If the time-average power of the incident wave is  $1 \text{ W/m}^2$ , what are the time-average powers of the transmitted and reflected waves?  
 (c) (5%) What are the polarization states of the transmitted and reflected waves? (linear, circular, or elliptic)



1. (20 %) The electric field inside a certain dielectric-slab waveguide (where  $\epsilon_d = 9 \epsilon_0$ ,  $\mu_d = \mu_0$ ) is given by

$$\vec{E} = \bar{y}10 \cos(5000x)e^{-j5000z} \text{ mV} \cdot \text{m}^{-1}$$

The thickness of the slab is  $d = 2 \text{ mm}$ . What is the waveguide wavelength  $\bar{\lambda}$  in meters? What is the wave frequency  $f$ ? Identify the propagating mode (i.e. specific whether it is TE or TM, odd or even, and find the value of the mode index number  $m$ ). Specify all other possible propagating modes at frequency  $f$ .

2. (15 %) A circular cavity resonator is designed to resonate in the  $\text{TE}_{011}$  mode over a frequency range of 8 GHz to 12 GHz. The tuning is accomplished by moving one end plate, which effectively changes the length of the resonator. If the radius of the circular resonator is 3 cm, calculate the range displacement of the movable end that is required to meet the design specifications.

TABLE 5.3.  $l$ th roots ( $s_{nl}$ ) of  $J'_n(\cdot) = 0$ .

| $l$ | $n$    |        |        |        |        |        |        |        |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|
|     | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7      |
| 1   | 3.832  | 1.841  | 3.054  | 4.201  | 5.317  | 6.416  | 7.501  | 8.578  |
| 2   | 7.016  | 5.331  | 6.706  | 8.015  | 9.282  | 10.520 | 11.735 | 12.932 |
| 3   | 10.173 | 8.536  | 9.969  | 11.346 | 12.682 | 13.987 | 15.268 | 16.529 |
| 4   | 13.324 | 11.706 | 13.170 | 14.586 | 15.964 | 17.313 | 18.637 | 19.942 |

3. (15 %) An air-filled coaxial line has thick walls made of copper. Its attenuation constant is  $\alpha_c = 0.003 \text{ np} \cdot \text{m}^{-1}$  at 300 MHz. (a) What is its attenuation constant at 600 MHz? (b) If all of its cross-sectional linear dimensions (i.e., inner and outer radii  $a$ ,  $b$ , and thickness  $d$  of the outer conductor) are doubled, what is the new value of  $\alpha_c$  at 300 MHz? (c) Suppose the original line is filled with a dielectric for which  $\epsilon' = 2 \epsilon_0$  and the loss tangent  $\tan \delta_c = (15/\pi) \times 10^{-4}$  at 300 MHz. Find the total attenuation constant constant  $(\alpha_c + \alpha_d)$  at 300 MHz.

- (20pts) A point charge  $q$  is constrained to the vicinity of the origin of a coordinate system by a driving force  $\mathbf{F} = -k\mathbf{x}$ , where  $k$  is a constant and  $\mathbf{x}$  is the displacement from the origin to the point where  $q$  is located. The charge  $q$  acts as a damped oscillator with a damped coefficient  $\kappa$ . (a) If an external electric field  $\mathbf{E} = E_0\hat{x}$  is applied, please write down the force equation of a driven damped oscillator. [4pts] (b) Let  $E = E_0e^{i\omega t}$  and  $x = x_0e^{i\omega t}$ , where  $E_0$  and  $x_0$  are real quantities. Substitute into the above equation and solve the displacement  $x$ . [3pts] (c) A polarization is defined as  $\mathbf{P} = \hat{x}Nqx = \epsilon_0\chi_e\mathbf{E}$ , where  $N$  is the volume density of charge and permittivity  $\epsilon_r = 1 + \chi_e$ . Substitute  $x$  into polarization to respectively solve the real and imaginary parts of  $\epsilon_r$ . [5pts] (d) Plot the real and imaginary parts of  $\epsilon_r(\omega)$ , and explain the significances of them. [8pts]
- (14pts) A silicon sample with unknown doping is tested in a Hall effect experiment with  $B_0 = 1\text{T}$ , as shown in Fig.1. The sample dimensions are  $L = 3\text{mm}$ ,  $w = 0.5\text{mm}$ , and  $d = 0.1\text{mm}$ . A dc voltage of  $V_0 = 100\text{mV}$  is applied, which results in a current flow of  $I_0 \sim 5\text{mA}$ . If a Hall voltage of  $V_H \sim 3\text{mV}$  is measured with its plus terminal being at the top terminal, find (a) the type of the semiconductor (n-type or p-type); (b) the approximate impurity concentration ( $\sim N_e$  or  $\sim N_p$ ); and (c) the mobility of the impurity carriers ( $\mu_e$  or  $\mu_p$ ).

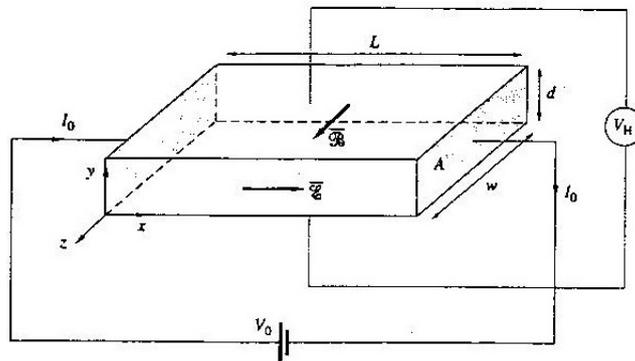


Fig.1

- (16pts) Define the following antenna parameters: (a) directive gain, (b) power gain, (c) effective area (or effective aperture), and (d) radiation resistance.