Part 1: 50 points, please describe your answers as complete as possible.

1. (10 points)
   (a) Write out Maxwell's equations in the differential form and indicate what's meaning for each item in equations?
   (b) According to Maxwell's equations for a simple, source-free, and lossless medium, show that both of electrical field and magnetic field are satisfied with individual wave equation.

2. (10 points)
   Consider the following complex phasor expression for a time-harmonic electric field in free space:

$$\vec{E} = \left[3\hat{x} + 4\hat{y} + j5\hat{z}\right]e^{-j(8x-6y)\pi} \qquad \text{mV-m}^{-1}$$

   (a) Is this a uniform plane wave? (b) What is the direction of propagation and the state of polarization of this electromagnetic field? (c) Find the associated magnetic field and the time-average power density in the direction of propagation.

3. (10 points)
   A beam of light is normally incident on one side of a 1-cm thick slab of flint glass (assume n=1.86) at 550 nm. (a) What percentage of the incident power reflects back? (b) To minimize reflections, the glass is coated with a thin layer of antireflection coating material on both sides. The material chosen is magnesium fluoride (MgF_2), which has a refractive index around 1.38 at 500 nm. Find the approximate thickness of each coating layer of MgF_2 needed.

4. (20 points)
   Assume a propagating plane EM wave along the +z-axis with angular frequency ω and wave number K where the medium possesses the following properties:

$$\begin{cases} \varepsilon=\varepsilon_0, \sigma=0, \mu=\mu_0, \text{ when } z<0 \\ \varepsilon>\varepsilon_0, \sigma>0, \mu=\mu_0, \text{ when } z>0 \end{cases}.$$

   (a) For this EM wave, what is the dispersion relation in lossy medium (z>0)? (b) For two cases of poor conductor and good conductor, show wave numbers of propagating EM waves? (c) What are skin depths in (b) cases? (d) If $\sigma << \varepsilon\omega$ is a condition in medium, calculate the transmittance for this EM wave passing through the medium?

Part 2: 50 points, please describe your answers as complete as possible.

1. (30 points)
   For an infinite dielectric (made of simple medium) slab waveguide of thickness *d* situated in air, answer the following questions about the instantaneous expressions of all nonzero field components, eigenvalue equation, and cutoff frequency for odd TM modes. Remember to show your detailed steps.
   (a) In a source-free environment, rewrite the real-field Maxwell's equations into complex-field Maxwell's equations for a time-harmonic electromagnetic wave.
   (b) Derive the inter-relationships among all complex-field components according to (a).
   (c) Derive a wave equation for the complex electric field based upon (b).
   (d) Assume the electromagnetic wave propagates along the z-direction in Cartesian coordinates. Based on (c), derive the z-components of the complex electric field in all regions for odd TM modes, where the boundary condition, the tangential component of the electric field must be continuous across the boundary, is used.
   (e) Based on (b) and (d), derive other complex-field components in all regions for odd TM modes.
   (f) Based on (e), obtain the instantaneous expressions of all the field components for odd TM modes in all regions.
   (g) Based on (e), derive the eigenvalue equation for odd TM modes by using the boundary condition, the tangential component of the magnetic field must be continuous across the boundary for non-conducting medium.
   (h) Based on (g), derive the cutoff frequency for odd TM modes.

2. (20 points)
   A certain microwave oven is designed from aluminum rectangular waveguide with dimensions a = 29.21 cm and b = 14.605 cm. The length of the guide is d with both open ends closed with aluminum sheets.
   (a) For $d = a$ = 29.21 cm, find the resonant frequencies of the two nontrivial TE_{mnp} and TM_{mnp} modes that have the lowest resonant frequencies.
   (b) Redesign the oven by readjusting the length d such that the resonant frequency of the TE_{101} modes is 1 GHz.
   (c) In part (b), what are the resonant frequencies of the TE_{102} and TE_{011} modes?

Part 3: 50 points, please describe your answers as complete as possible.

1. Consider a conductor with a cloud of freely circulating electrons under equilibrium, which was driven by an electric field $E= E_0\ ^{-i\omega t}$ and the electrons have damped harmonic oscillations. Suppose a single electron with charge of q_e and mass of m_e, and the damping friction force is proportional to $m\tau^{-1}\mathrm{v_d}$, wherein $\mathrm{v_d}$ is the drift velocity of electrons.
   (a) (6 points) Please write down the force equation of a driven damped oscillator and explain the significance of each term.
   (b) (6 points) Let $x=x_0\ e^{-i\omega t}$, substitute into the above expression and show that $x = \frac{q_e E}{m_e}\frac{1}{\omega^2 + i\tau^{-1}\omega}$.
   (c) (8 points) Show that the effective permittivity $\varepsilon_{eff}(\omega) = [1 - \frac{\omega_p}{\omega^2}]$, wherein $\omega_p = \sqrt{Nq_e^{\ 2} / \varepsilon_0 m}_e$ is the plasma frequency and N is the volume density of electrons.

2. (20 points)
   Show that (a) the directive gain of the Hertzian dipole is $g_d(\theta,\phi) = 1.5\sin^2\theta$ and (b) the directive gain of the half-wave dipole is $g_d(\theta,\phi) = 1.64\frac{\cos^2(\frac{\pi}{2}\theta)}{\sin^2\theta}$.

3. (10 points)
   Find the maximum effective area of a λ/2 wire dipole operating at 30 MHz. How much power is received with an incident plane wave of strength 2 mV/m?