Part I (50 points)

1. The electric filed intensity for a transverse electromagnetic wave can be represented as $\vec{E}(\vec{R}) = \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}}$, where $\vec{R}$ is the radius vector from the origin, $\vec{k}$ is the wavenumber vector, and $\vec{E}_0$ is a constant vector.

   (a) Explain that the wave is a uniform plane wave. (5 pts)

   (b) Prove that the intensity satisfies the equations of $\vec{a}_n \cdot \vec{E}_0 = 0$ and $\vec{H}(\vec{R}) = (1/\eta)\vec{a}_n \times \vec{E}(\vec{R})$, where $\vec{a}_n$ is a unit vector in the direction of propagation and η is the intrinsic impedance of the medium. (10 pts)

2. (a) Write the definition of the Poynting vector and explain the physical meaning of the vector. (3 pts)
   (b) Write Poynting’s theorem and drive the theorem beginning from Maxwell’s equations. (12 pts)

3. Prove that, under the condition of no reflection at an interface, the sum of the Brewster angle and the angle of refraction is π/2 for parallel polarization ($\varepsilon_1 \neq \varepsilon_2$). (10 pts)

4. Prove that an elliptically polarized plane wave can be resolved into right-hand and left-hand circularly polarized waves. (10 pts)

Part II (50 points)

1. **Dielectric Waveguide(30 pts)**

TM modes propagate in the z-direction of a dielectric slab waveguide with dielectric material ε_d of thickness d surrounding by air ε_0.

From $\frac{d^2}{dx^2}E_z^0 + h^2 E_z^0 = 0$ where $h^2 = \gamma^2 + k^2 = (j\beta)^2 + \omega^2\mu\varepsilon$,

E_z has a form $E_z^0(x) = E_0 \sin(k_y x) + E_e \cos(k_y x)$ $|x| \le \frac{d}{2}$ inside the dielectric slab

and $E_z^0(x) = \begin{cases} C_u e^{-\alpha\left(x-\frac{d}{2}\right)} & x \ge \frac{d}{2} \\ C_l e^{+\alpha\left(x+\frac{d}{2}\right)} & x \le \frac{d}{2} \end{cases}$ outside the dielectric slab waveguide

(a) (5 pts) Find $k_x^2 + \alpha^2 = ?$

(b) (10 pts)Using the boundary condition at x=d/2, -d/2 for E_z, write down the **expression** of E_z^0 E_x^0 H_y^0 both inside (-d/2< x < d/2) and outside (|x| >d/2) for odd TM mode and even TM mode. With H_y boundary condition at x=d/2, -d/2, find the **dispersion relation** for both odd and even TM mode.

(c) (5 pts) What is the **cutoff frequency** for odd TM_1 and even TM_1 mode for d=1μm and ε_d =4 ($1/\sqrt{\mu_0\varepsilon_0} = c = 3\times10^8 m/s$)

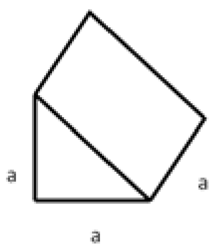
(d) (5 pts) Use the ray approach to determine the **propagation angle** of odd TM_1 mode in case (c)

(e) (5 pts) if we put a **PEC** plate at x=0 , what are the cut-off frequency of the first f_{c1} and second f_{c2} modes for case (c)

2. **Resonators (20 pts)**

(a)(10 pts) Determine the **dominant mode** and their **resonant frequencies** in an-air-filed rectangular cavity metal resonator for (I) a>b>d, (II) a>d>b, and (III) a=b=d, where a, b, and d are the dimensions in the x-, y-, and z-directions, respectively. And does TE_{mnp} or TM_{mnp} resonant mode have **larger loss** if the metal wall has Ohm loss with conductivity σ ? Please explain your answer.

(b) (10 pts) If the **cubic** cavity resonator with size **a** is cut by half as a **right triangular** cavity resonator. What is the resonate frequency of the first mode? (Hint: this can be considered as a superposition of two rectangular cavity modes, such that all the parallel E fields at the edges of the triangle vanish.)



Part III. (50 points)

1. For the Hertzian dipole, we know that magnetic field is

$$\vec{H} = -\hat{\varphi}\frac{I\cdot dl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}\right]e^{-j\beta r}$$

   Please derive the complex Poynting vector $\vec{S}$ (15 pts) and the total time-average power radiated by the antenna (10 pts).

2. Please derive the frequency response of dielectric materials. (Hint: Assume the external field and the motion of electron are both in the x direction).
   (a) Write down the differential equation for the motion of electron (10 pts).
   (b) Solve the equation (10 pts).
   (c) Derive the dielectric function of the materials (5 pts).