博士班 104 學年度第 2 學期資格考 電磁理論 科試題

Part I

1.

(a) (10%) Consider a source-free and lossless medium; derive the homogeneous vector Helmholtz's equation for **E** from the Maxwell's equations

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$
$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{H} = 0$$

(b) (5%)Calculate the wave number k associated with the wave described by the Helmholtz's equation in (a).

(c) (5%) Now, consider a uniform plane wave characterized by a uniform E_x, write the one-dimensional wave equation for E_x.

(d) (5%) The solution of the wave equation in (b) should represent waves propagating in the +z and –z directions. What is E_x?

(e) (5%) What is the velocity of the propagating wave?

2.

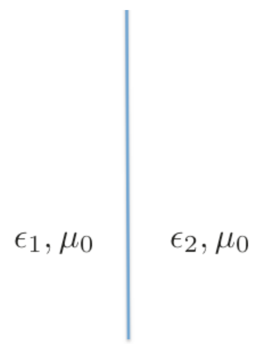
Suppose the electric field and the corresponding magnetic field associated with a plane sinusoidal electromagnetic wave are

$$\mathbf{E} = E_0 cos(kx - \omega t)\mathbf{y}$$
$$\mathbf{H} = B_0 cos(kx - \omega t)\mathbf{z}$$

(10%) Calculate the rate of energy flow per unit area for the wave.

3.

(10%) Derive the Brewster angle for a plane wave incident obliquely on a plane dielectric boundary (parallel polarization)



Part II

1. (20%) A waveguide is formed by two parallel copper sheets with σ_c = 5.8 x 10^7 (S/m) separated by a 5-cm thick lossy dielectric with ε_r = 2.25, μ_r = 1, and σ = 10^{-10} (S/m). For an operating frequency of 10GHz, find β, attenuation constant of dielectric α_d, attenuation constant of conductor α_c, phase velocity u_p, group velocity u_g, and guide wavelength λ_g for (a) TEM mode, (b) the TM_1 mode, and (c) the TE_1 mode.
2. (20%) An average power of 1 kW at 10GHz is to be delivered to an antenna at the TE10 mode by an air filled rectangular copper with σ_c = 5.8 x 10^7 (S/m) waveguide 1 m long and having sides a = 2.25 cm and b = 1 cm. Please find
   (a) the attenuation constant due to conductor losses,
   (b) the maximum values of the electric and magnetic field intensities within the waveguide,
   (c) the maximum value of the surface current density on the conducting walls,
   (d) the total amount of average power dissipated in the waveguide.
3. (10%) An air filled rectangular cavity with brass walls (ε_0, μ_0, σ = 1.57 x 10^7 (S/m)) has the following dimensions: a = 4 cm, b = 3 cm, and d = 5 cm.
   (a) Determine the dominant mode and its resonant frequency for this cavity.
   (b) Find the Q and time average stored electric and magnetic energies at the resonant frequency, assume H_0 to be 0.1 A/m

Parameters: ε_0 = (1/36π) x 10^{-9} F/m, μ_0 = 4π x 10^{-7} H/m, c = 3 x 10^8 m/s.

Part III

1. For a Hertzian dipole, mathematically derive its near-zone fields and far-zone fields. Remember to show your detailed steps (25%)

2. The electrostatic deflection system of a cathode-ray oscilloscope is shown below. Electrons from a heated cathode are given an initial velocity $\vec{u}_0 = \hat{a}_z u_0$ by a positively charged anode. The electron enters at $z = 0$ into a region of deflection plates where a uniform electric field $\vec{E}_d = -\hat{a}_y E_d$ is maintained over a width w. (25%)

   (a) Ignoring gravitational effects, find the vertical deflection of the electrons on the fluorescent screen at $z = L$.

   (b) Find the relation between the angle of arrival, α, of the electron beam at the screen and the deflecting electric field intensity E_d.

   (c) Find the relation between w and L such that $d_1=d_0/20$.

   (d) Assuming no break down voltage in insulation, what is the maximum voltage that can be measured if the distance of separation between the plates is h?

   (e) What is the restriction on L if the diameter of the screen is D?